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THE EXTERNAL NEUMANN PROBLEM FOR A SEMI-INFINITE CYLINDER*

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Neumann problem is considered in a region external to a semi-infinite cylinder. Boundary value problems are formulated for the coefficients of expansion of the solution into a Fourier series in azimuthal variables, and the method of dual integral equations is applied to these problems. It is shown that the integral Fredholm equation obtained in this manner have unique continuous solutions, which can be obtained using the iterative methods. The theory developed here is illustrated by the problem of a potential flow around a cylinder.

1. Formulation of the problem and its reduction to dual integral equations. We require to find a solution of the Laplace equation

$$\Delta u = 0 \tag{1.1}$$

in the region outside a semi-infinite circular cylinder of unit radius. The solution should vanish at infinity, and satisfy the following conditions at the cylinder surface:

$$\left.\frac{\partial u}{\partial z}\right|_{z=0,\, r<1} = f(r, \varphi), \quad \left.\frac{\partial u}{\partial r}\right|_{r=1,\, z>0} = g(z, \varphi) \tag{1.2}$$

where f and g are given functions and r, φ, z are cylindrical coordinates.

Expanding the solution into a Fourier series in terms of the azimuthal angle

$$u = \frac{1}{2} u_0^c(r, z) + \sum_{n=1}^{\infty} [u_n^c(r, z) \cos n\varphi + u_n^s \sin n\varphi] \tag{1.3}$$

we obtain the following boundary value problems for the coefficients of the expansion:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_n}{\partial r}\right) + \frac{\partial^2 u_n}{\partial z^2} - \frac{n^2}{r^2} u_n = 0, \qquad \left.\frac{\partial u_n}{\partial z}\right|_{z=0,\, r<1} = f_n(r), \quad \left.\frac{\partial u_n}{\partial r}\right|_{r=1,\, r>0} = g_n(z) \tag{1.4}$$

$$\begin{pmatrix} f_n^c(r) \\ f_n^s(r) \end{pmatrix} = \frac{1}{\pi}\int_0^{2\pi} f(r, \varphi) \begin{pmatrix} \cos n\varphi \\ \sin n\varphi \end{pmatrix} d\varphi, \quad \begin{pmatrix} g_n^c(z) \\ g_n^s(z) \end{pmatrix} = \frac{1}{\pi}\int_0^{2\pi} g(z, \varphi) \begin{pmatrix} \cos n\varphi \\ \sin n\varphi \end{pmatrix} d\varphi$$

where u_n, f_n and g_n denote either u_n^s, f_n^s and g_n^s, or u_n^c, f_n^c and g_n^c.

To solve the boundary value problem (1.4), we divide the outside of the cylinder into two regions I $(z<0,\ 0 \leqslant r < \infty)$ and II $(z>0,\ 1<r<\infty)$, and write the functions $u_n(r, z)$ in these regions in the form of integral expansions

$$u_n(r, z) = \int_0^{\infty} A_n(\lambda) J_n(\lambda r) e^{\lambda z} d\lambda \quad (z<0, 0 \leqslant r < \infty) \tag{1.5}$$

$$u_n(r, z) = \frac{2}{\pi}\int_0^{\infty} \frac{K_n(\nu r)}{K_n'(\nu)} \frac{\cos \nu z}{\nu} d\nu \int_0^{\infty} g_n(\zeta) \cos \nu\zeta d\zeta + \int_0^{\infty} B_n(\lambda) \operatorname{Im}\left\{\frac{H_n^{(1)}(\lambda r)}{[H_n^{(1)}(\lambda)]'}\right\} e^{-\lambda z} d\lambda \quad (z>0, r>1)$$

Here J_n and $H_n^{(1)}$ are Bessel functions of first and third kind, and K_n is a Bessel function of an imaginary argument.

If $f(r, \varphi)$ and $g(z, \varphi)$ satisfy the Dirichlet conditions with respect to all variables and the integral of $g(z, \varphi)$ from zero to infinity converges absolutely for a fixed φ, then the functions sought decreases at infinity and the boundary condition at the side surface of the cylinder holds. Satisfying the boundary condition at the end face of the cylinder and demanding that the function sought as well as its derivative in z are both continuous in the plane $z=0$ when $r>1$, enable us to reduce the problem to that of solving the following dual integral equations:

$$\int_0^{\infty} \lambda A_n(\lambda) J_n(\lambda r) d\lambda = f_n(r), \quad 0 \leqslant r < 1, \qquad \int_0^{\infty} A_n(\lambda) J_n(\lambda r) d\lambda = F_n(r), \quad r>1 \tag{1.6}$$

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$$F_n(r) = h_n(r) + \int_0^\infty B_n(\lambda)\,\mathrm{Im}\,\frac{H_n^{(1)}(\lambda r)}{[H_n^{(1)}(\lambda)]'}\,d\lambda, \qquad h_n(r) = \frac{2}{\pi}\int_0^\infty \frac{K_n(\nu r)}{K_n'(\nu)}\,\frac{\partial\nu}{\nu}\int_0^\infty g_n(\zeta)\cos\nu\zeta\,d\zeta$$

The following auxilliary integral relation holds for $A_n(\lambda)$ and $B_n(\lambda)$

$$\int_0^\infty \lambda A_n(\lambda) J_n(\lambda r)\,d\lambda + \int_0^\infty \lambda B_n(\lambda)\,\mathrm{Im}\,\frac{H_n^{(1)}(\lambda r)}{[H_n^{(1)}(\lambda)]'}\,d\lambda = 0, \quad r > 1 \tag{1.7}$$

2. Solution of dual integral equations. We assume that the function $F_n(r)$ is given. Then a solution of the pair of equations (1.6) can be found e.g. as follows. The known formulas of the theory of Bessel functions /1/ show that the operators

$$\Lambda_1[u(r)] = \sqrt{\frac{2}{\pi t}}\,t^{-n}\int_0^t \frac{r^{n+1}u(r)\,dr}{\sqrt{t^2-r^2}}, \qquad \Lambda_2[u(r)] = -\sqrt{\frac{2}{\pi t}}\,t^n\,\frac{d}{dt}\int_t^\infty \frac{r^{1-n}u(r)\,dr}{\sqrt{r^2-t^2}}$$

enable us, when applied to both parts of the equations (1.6) respectively, to reduce these dual integral equations to a single integral Hankel equation

$$\int_0^\infty \sqrt{\lambda}\,A_n(\lambda) J_{n+1/2}(\lambda t)\,d\lambda = \Phi_n(t), \quad t<1, \quad \Phi_n(t) = \Lambda_1[f_n(r)], \quad t>1, \quad \Phi_n(t) = \Lambda_2[F_n(r)] \tag{2.1}$$

Inversion of the formula (2.1)

$$A_n(\lambda) = \sqrt{\lambda}\int_0^\infty t\Phi_n(t) J_{n+1/2}(\lambda t)\,dt \tag{2.2}$$

yields a solution of the problem, provided that the functions $f_n(r)$ and $F_n(r)$ are both known.
In the present case however, the function $F_n(r)$ is not known, therefore the function $\Phi_n(t)$ is also unknown in the interval $1 < t < \infty$. Let us denote $\varphi_n(t) = \Phi_n(t)$ for $1 < t < \infty$. Relations (1.6) and (2.1) together with the known relations for the functions $H_\nu^{(1)}$, yield the following integral equation for $\varphi_n(t)$:

$$\varphi_n(t) = p_n(t) + \int_0^\infty \sqrt{\lambda}\,B_n(\lambda)\,\mathrm{Im}\,\frac{H_{n+1/2}^{(1)}(\lambda t)}{[H_n^{(1)}(\lambda)]'}\,d\lambda, \qquad p_n(t) = \Lambda_2[h_n(r)] \tag{2.3}$$

Formula (1.7) makes it possible to express the function $B_n(\lambda)$ in terms of $A_n(\lambda)$. To do this it is sufficient to use the Weber—Orr inversion theorem /2/

$$B_n(\lambda) = -\int_1^\infty \rho\,\mathrm{Im}\,\{[H_n^{(2)}(\lambda)]' H_n^{(1)}(\lambda\rho)\}\,N(\rho)\,d\rho, \qquad N(\rho) = \int_0^\infty \nu A_n(\nu) J_n(\nu\rho)\,d\nu$$

Relations (2.1) and (2.2) show that the function $A_n(\lambda)$ can be written as two terms, one of them known, and the other expressed in terms $\varphi_n(t)$

$$A_n(\lambda) = a_n(\lambda) + \sqrt{t}\int_1^\infty t\varphi_n(t) J_{n+1/2}(\lambda t)\,dt, \qquad a_n(\lambda) = \sqrt{\frac{2\lambda}{\pi}}\int_0^1 t^{-n+1/2} J_{n+1/2}(\lambda t)\,dt\int_0^t \frac{r^{n+1} f_n(r)}{\sqrt{t^2-r^2}}\,dr \tag{2.4}$$

Consequently, the function $B_n(\lambda)$ can be written in the form of a sum

$$B_n(\lambda) = b_n(\lambda) + C_n(\lambda), \quad b_n(\lambda) = -\int_1^\infty \rho\,\mathrm{Im}\,\{[H_n^{(2)}(\lambda)]' H_n^{(1)}(\lambda\rho)\}\,d(\rho)\,d\rho, \quad d(\rho) = \int_0^\infty \nu a_n(\nu) J_n(\nu\rho)\,d\nu \tag{2.5}$$

$$C_n(\lambda) = -\int_1^\infty \rho\,\mathrm{Im}\,\{[H_n^{(2)}(\lambda)]' H_n^{(1)}(\lambda\rho)\}\,T(\rho)\,d\rho, \qquad T(\rho) = \int_0^\infty \nu\sqrt{\nu}\,J_n(\nu\rho)\,d\nu\int_1^\infty \tau\varphi_n(\tau) J_{n+1/2}(\nu\tau)\,d\tau$$

Here again, the first term $b_n(\lambda)$ is known, and the second term can be written in terms of the function $\varphi_n(t)$. From (2.5) and (2.3) we obtain an integral equation for $\varphi_n(t)$

$$\varphi_n(t) = q_n(t) + \int_0^\infty \sqrt{\lambda}\,C_n(\lambda)\,\mathrm{Im}\,\frac{H_{n+1/2}^{(1)}(\lambda t)}{[H_n^{(1)}(\lambda)]'}\,d\lambda, \qquad q_n(t) = p_n(t) + \int_0^\infty \sqrt{\lambda}\,b_n(\lambda)\,\mathrm{Im}\,\frac{H_{n+1/2}^{(1)}(\lambda t)}{[H_n^{(1)}(\lambda)]'}\,d\lambda \tag{2.6}$$

Changing the order of integration in the last formula of (2.5), integrating by parts and applying the following relations of the theory of Bessel functions:

$$\nu J_n(\nu\rho) = \rho^{-n-1} d\,[\rho^{n+1} J_{n+1}(\nu\rho)]/d\rho, \qquad \sqrt{\frac{\pi}{2}} \int_0^\infty J_{n+1}(\nu\rho) J_{n+1/2}(\nu\tau) \sqrt{\nu\tau}\, d\nu = E_n(\rho, \tau)$$

$$\rho > \tau, \quad E_n = \tau^{n+1}\rho^{-n-1}/\sqrt{\rho^2-\tau^2}, \quad \rho < \tau, \quad E_n = 0, \quad d\,[\rho^{-n} H_n^{(1)}(\lambda\rho)]/d\rho = -\lambda\rho^{-n} H_{n+1}^{(1)}(\lambda\rho)$$

$$\int_\tau^\infty \frac{\rho^{-n} H_{n+1}^{(1)}(\lambda\rho)}{\sqrt{\rho^2-\tau^2}}\, d\rho = \sqrt{\frac{\pi}{2\lambda\tau}}\, \tau^{-n} H_{n+1/2}^{(1)}(\lambda\tau), \qquad \operatorname{Im} \frac{H_{n+1/2}^{(1)}(\lambda t)}{[H_n^{(1)}(\lambda)]'} \operatorname{Im} \{[H_n^{(2)}(\lambda)]' H_{n+1/2}^{(1)}(\lambda\tau)\} \equiv$$

$$J_{n+1/2}(\lambda t) J_{n+1/2}(\lambda\tau) - \operatorname{Re}\left\{ \frac{J_n'(\lambda)}{[H_n^{(1)}(\lambda)]} H_{n+1/2}^{(1)}(\lambda t) H_{n+1/2}^{(1)}(\lambda\tau) \right\}$$

as well as the Hankel expansion theorem /1/

$$\int_0^\infty \lambda J_{n+1/2}(\lambda t)\, d\lambda \int_1^\infty \tau\varphi_n(\tau) J_{n+1/2}(\lambda\tau)\, d\tau = \varphi_n(t)$$

we transform (2.6) to the form

$$\varphi_n(t) = \frac{1}{2} q_n(t) + \int_1^\infty M_n(t, \tau)\varphi_n(\tau)\, d\tau \tag{2.7}$$

$$M_n(t, \tau) = \frac{\tau}{2} \operatorname{Re} \int_0^\infty \lambda \frac{J_n'(\lambda)}{[H_n^{(1)}(\lambda)]'} H_{n+1/2}^{(1)}(\lambda t) H_{n+1/2}^{(1)}(\lambda\tau)\, d\lambda = \frac{\tau}{\pi} \int_0^\infty \frac{I_n'(x)}{K_n'(x)} K_{n+1/2}(xt) K_{n+1/2}(x\tau)\, x dx$$

The last transformation was carried out, just as in /3/, with help of the Cauchy theorem.

Equation (2.7) represents a Fredholm equation of second kind with kernel $M_n(t, \tau)$, which can be expressed in terms of the quadratures of special functions.

Let us introduce a new unknown function

$$\psi_n(x) = \frac{I_n'(x)\sqrt{x}}{K_n'(x) I_n(x)} \int_1^\infty \tau\varphi_n(\tau) K_{n+1/2}(x\tau)\, d\tau \tag{2.8}$$

The kernel of the corresponding integral equation is written for the function $\psi_n(x)$ in a closed form. Indeed, taking into account (2.8) we can write (2.7) in the form

$$\varphi_n(t) = \frac{1}{2} q_n(t) + \frac{1}{\pi} \int_0^\infty \sqrt{x}\, K_{n+1/2}(xt) I_n(x) \psi_n(x)\, dx \tag{2.9}$$

From (2.9) and (2.8) we obtain an integral equation for the function $\psi_n(x)$

$$\psi_n(x) = S_n(x) + \int_0^\infty L_n(x, y)\psi_n(y)\, dy, \qquad S_n(x) = \frac{I_n'(x)\sqrt{x}}{2K_n'(x) I_n(x)} \int_1^\infty q_n(\tau)\, \tau K_{n+1/2}(x\tau)\, d\tau \tag{2.10}$$

$$L_n(x, y) = \frac{1}{\pi} \frac{I_n'(x)\sqrt{xy}\, I_n(y)}{K_n'(x) I_n(x)} \int_1^\infty \tau K_{n+1/2}(x\tau) K_{n+1/2}(y\tau)\, d\tau =$$

$$\frac{1}{\pi} \frac{I_n'(x)\sqrt{xy}\, I_n(y)}{K_n'(x) I_n(x)} \frac{x K_{n-1/2}(x) K_{n+1/2}(y) - y K_{n-1/2}(y) K_{n+1/2}(x)}{x^2 - y^2}$$

We shall show that equation (2.10) has a unique solution in the class of functions continuous and bounded at infinity. To do this (see e.g. /3,4/), it is sufficient to confirm that

$$\int_0^\infty |L_n(x, y)|\, dy < 1 \tag{2.11}$$

Using the known integrals /1/

$$\int_0^\infty I_n(y) K_{n+1/2}(yt) \sqrt{yt}\, dy = \sqrt{\frac{\pi}{2}} \frac{t^{-n}}{\sqrt{t^2-1}}, \qquad \int_1^\infty \frac{\sqrt{t}\, K_{n+1/2}(xt)}{t^n \sqrt{t^2-1}}\, dt = \sqrt{\frac{\pi}{2x}} K_n(x)$$

we obtain

$$\int_0^\infty |L_n(x, y)|\, dy = -\frac{1}{2} \frac{I_n'(x) K_n(x)}{K_n'(x) I_n(x)}$$

and from the formula /5/

$$I_n(x) K_n(x) = \int_0^\infty \frac{t}{t^2 + x^2} J_n^2(t)\, dt$$

it follows that the product $I_n(x) K_n(x)$ is a decreasing function of its argument, i.e. $I_n'(x) K_n(x) / (I_n(x) K_n'(x)) < 1$. This implies the inequality (2.11) and hence the uniqueness of the solution of (2.10).

3. **Potential flow past a semi-infinite cylinder.** To illustrate the theory constructed in Sects.1 and 2, we consider a problem of potential flow of fluid past a semi-infinite cylinder, constant at infinity and inclined at the angle α to the cylinder axis. Let $u(r, \varphi, z)$ be the velocity potential of the problem, i.e. $\mathbf{v} = \operatorname{grad} u$. The Fourier series (1.3) consists in this case of two terms only, since the potential u_0 of the velocities v_0 assumes the following form at infinity:

$$u_0 = v_0 z \cos\alpha - v_0 r \sin\alpha \cos\varphi$$

To separate the singularities at infinity, we write the solution of the problem in the form

$$u(r, \varphi, z) = v_0 \cos\alpha\, [z - u_0(r, z)] - v_0 \sin\alpha \cos\varphi\, [r - u_1(r, z)], \quad z < 0 \tag{3.1}$$

$$u(r, \varphi, z) = v_0 \cos\alpha\, [z - u_0(r, z)] - v_0 \sin\alpha \cos\varphi\, [(r + r^{-1}) - u_1(r, z)], \quad z > 0$$

and this enables us to assume that the unknown functions $u_0(r, z)$ and $u_1(r, z)$ decrease at infinity.

Taking now into account the condition of impermeability of the side surface of the cylinder, we write the functions u_0 and u_1 in the form of expansions (1.5), putting in the last equation $g_n(\zeta) = 0$. The condition of impermeability of the end face of the cylinder and the continuity of the function u and of its derivative in z in the plane $z = 0$ for $r > 1$, yields two pairs of integral equations (1.6) in which

$$f_0(r) = 1, \quad h_0(r) = 0, \quad f_1(r) = 0, \quad h_1(r) = -1/r \tag{3.2}$$

and two sets of conditions (1.7) for $n = 0$ and $n = 1$.

Equations (2.10) for $n = 0, 1$ have the form

$$\psi_0(x) = R_0(x) \left[\int_0^\infty \frac{y+1}{y^2} \frac{I_1(y)}{K_1(y)} e^{-2y} \frac{dy}{x+y} + \int_0^\infty \frac{I_0(y)}{x+y} e^{-y} \psi_0(y)\, dy \right] \tag{3.3}$$

$$\psi_1(x) = R_1(x) \left[-\frac{\pi}{2x} + \int_0^\infty I_1(y) e^{-y} \left(\frac{1}{x+y} + \frac{1}{xy} \right) \psi_1(y)\, dy \right], \qquad R_n(x) = \frac{I_n'(x)}{2K_n'(x) I_n(x)} e^{-x}$$

The kernels of these equations can be obtained directly from the last formula of (2.10), by putting $n = 0, 1$. To find the free term in the second equation of (3.3), we must remember that, according to (3.2), $f_1(r) = 0$, consequently in the second formula of (2.4) $a_1(\lambda) = 0$. Therefore in (2.5) $b_1(\lambda) = 0$ and this implies that $q_1(t) = p_1(t)$ (see (2.6)). Since by virtue of (3.2) $h_1(r) = -r^{-1}$, therefore from (2.3) follows

$$p_1(t) = \sqrt{\frac{2t}{\pi}} \frac{d}{dt} \int_t^\infty \frac{dr}{r\sqrt{r^2 - t^2}} = -\sqrt{\frac{\pi}{2t^3}}$$

Using this together with the second equation of (2.10), we obtain the integral for $n = 1$ to obtain $S_1(x) = -\pi R_1(x) / (2x)$ which corresponds to the free term in the second equation of (3.3). The free term in the first equation of (3.3) is found in the same manner. According to (2.4) we have, for $n = 0$ and $f_0(r) = 1$,

$$a_0(\lambda) = 2\pi^{-1} \operatorname{Im}\, [(1 - i\lambda)\, e^{i\lambda} / \lambda^2] \tag{3.4}$$

From (2.5) we obtain

$$b_0(\lambda) = 2\pi^{-1} \operatorname{Re}\, [(i\lambda - 1)\, H_1^{(2)}(\lambda)\, e^{i\lambda} / \lambda^2] \tag{3.5}$$

Since according to (3.2) $p_0(t)$ is identically zero, (2.6) yields, after a series of transformations,

$$q_0(t) = \sqrt{\frac{2}{\pi t}} \int_0^\infty \frac{x+1}{x^2} \frac{I_1(x)}{K_1(x)} e^{-(t+1)x}\, dx \tag{3.6}$$

Substituting (3.6) into the second equation of (2.10), we obtain the expression for $S_0(x)$ in the first equation of (3.3).

Let us introduce new unknown functions

$$\omega_0(x) = \frac{(x+1)\,I_1(x)}{x^2 K_1(x)}\,e^{-2x} + I_0(x)\,e^{-x}\psi_0(x), \qquad \omega_1(x) = I_1(x)\,e^{-x}\psi_1(x) \tag{3.7}$$

Then equations (3.3) will assume the form

$$\omega_0(x) = \frac{I_1(x)}{2K_1(x)}\,e^{-2x}\left[\frac{2(x+1)}{x^2} - \int_0^\infty \frac{\omega_0(y)\,dy}{x+y}\right], \qquad \omega_1(x) = -\frac{I_1'(x)}{2K_1'(x)}\,e^{-2x}\left[\frac{\pi}{2x} - \int_0^\infty\left(\frac{1}{x+y} + \frac{1}{xy}\right)\omega_1(y)\,dy\right] \tag{3.8}$$

Let us now write the coefficients $A_n(\lambda)$ and $B_n(\lambda)$ sought, in terms of $\omega_n(x)$ $(n = 0,1)$. Taking into account (3.4) and (3.5) and the last relation of (2.5), we can write (2.4) and (2.5) in the form

$$A_n(\lambda) = \operatorname{Re}\left[\beta_n(\lambda) + \sqrt{\lambda}\int_1^\infty t\varphi_n(t)\,H^{(1)}_{n+1/2}(\lambda t)\,dt\right], \qquad B_n(\lambda) = -\operatorname{Im}\left\{[H_n^{(2)}(\lambda)]'\left[\beta_n(\lambda) + \sqrt{\lambda}\int_1^\infty t\varphi_n(t)\,H^{(1)}_{n+1/2}(\lambda t)\,dt\right]\right\} \tag{3.9}$$

$$\beta_0(\lambda) = 2\pi^{-1}i\,(i\lambda - 1)\,e^{i\lambda}/\lambda^2, \quad \beta_1(\lambda) = 0$$

Expressions (3.9) show that the coefficients of the expansions (1.5) in the case of the flow in question are given in terms of a single function of λ. Putting

$$D_n(\lambda) = \beta_n(\lambda) + \sqrt{\lambda}\int_1^\infty t\varphi_n(t)\,H^{(1)}_{n+1/2}(\lambda t)\,dt \tag{3.10}$$

we obtain

$$A_n(\lambda) = \operatorname{Re} D_n(\lambda), \quad B_n(\lambda) = -\operatorname{Im}\{[H_n^{(2)}(\lambda)]' D_n(\lambda)\} \tag{3.11}$$

From (2.9) and (3.7) it follows that the function $\varphi_n(t)$ can be written in terms of $\omega_n(t)$ in the form

$$\varphi_0(t) = \sqrt{\frac{1}{2\pi t}}\int_0^\infty \omega_0(x)\,e^{-(t-1)x}\,dx, \qquad \varphi_1(t) = -\frac{1}{2}\sqrt{\frac{\pi}{2}}\,t^{-3/2} + \frac{1}{\pi}\int_0^\infty \sqrt{x}\,K_{3/2}(xt)\,e^{x}\omega_1(x)\,dx$$

Subsituting these expressions into (3.10), we obtain

$$D_0(\lambda) = \frac{i}{\pi}\,e^{i\lambda}\left[2\,\frac{i\lambda - 1}{\lambda^2} + \int_0^\infty \frac{\omega_0(x)}{i\lambda - x}\,dx\right], \qquad D_1(\lambda) = \frac{1}{\pi}\,e^{i\lambda}\left[-\frac{\pi}{2i\lambda} + \int_0^\infty\left(\frac{1}{i\lambda - x} + \frac{1}{i\lambda x}\right)\omega_1(x)\,dx\right] \tag{3.12}$$

Thus, having found the functions $\omega_n(t)$ from the integral equations (3.8) and having computed $D_n(\lambda)$ with help of (3.12), we obtain from (3.11) $A_n(\lambda)$ and $B_n(\lambda)$, and hence the functions $u_0(r, z)$ and $u_1(r, z)$. This solves, in accordance with (3.1), the problem of the flow in question.

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